**MINISTRY OF EDUCATION**

**RUHANGO DISTRICT**

**MATHEMATICS EXAM OF THE THIRD TERM 2021 FOR S3**

**DURATION: 3 Hours**

**INSTRUCTIONS:**

1. This paper has **TWO**sections: **A** and **B**

 **SECTION A:** Attempt **ALL** questions **(55 MARKS)**

 **SECTION B:** Attempt **ANY THREE** questions **(45MARKS)**

1. **Use of non-programmable calculators is** allowed.
2. Use only **a blue or black ink pen** to write your answers
3. Show clearly all the working steps. Marks will **not be awarded for the answer without all working steps.**

**SECTION A (ANSWER ALL QUESTIONS)**

1. Without using calculator, evaluate:

$\frac{3\frac{1}{3} - 1\frac{5}{6} × \frac{3}{11}}{1\frac{3}{4} + 7\frac{2}{3} ÷3\frac{5}{6}}=$**/3marks**

1. a) Find the ratio $x :y$ given that $\left(2x^{2 }+ 5y^{2}\right)÷y^{2}= \left(12x +y\right) ÷ 2y$

 b) Find the integral values of $x$ which satisfy the simultaneous inequalities

$3x -2 <10 +x \leq 2 +5x$**/4marks**

1. Simplify  Leaving your answer in surds with rational

denominator. **/3marks**

1. Solve the following system of two equations using the graphical method:

$\left\{\begin{array}{c}x+2y=4\\-x+2y=8\end{array}\right.$ **/3marks**

1. Find the solution set for $0\leq 2x-3 \leq x+8 $
2. If $x$ is a prime number
3. If $x$ is an even integer.**/4marks**
4. Evaluate the following giving your answer in the base five.

$454\_{six}+ 833\_{nine}- 221\_{three}$**/3marks**

1. 100 000 RWF was invested. The simple interest after two years was 16 000 RWF. Find the annually rate**/3marks**
2. Given that $x $and $y$ are proportional to 6 and 4 and that$2x +3y =72$. Determine the values of $x$and$ y$. **/4marks**
3. Consider the polynomial $P\left(x\right)=x^{4}-\left(1+\sqrt{2}\right)x^{3}+\left(1+\sqrt{2}\right)x^{2}-\left(1+\sqrt{2}\right)x+\sqrt{2}$
4. Calculate$P(\sqrt{2})$.

Divide $P(x)$ by $x-\sqrt{2}$**/5marks**

1. Given the vectors $\vec{a } = \left(\genfrac{}{}{0pt}{}{3}{-2}\right)$ , $\vec{b }= \left(\genfrac{}{}{0pt}{}{-1}{2}\right)$ and $\vec{c }= \left(\genfrac{}{}{0pt}{}{-4}{2}\right)$ . Find:
2. $\left‖ 3\vec{a }\right.- 4\vec{b }+ \frac{1}{2}\left.\vec{c }\right‖$
3. Find the coordinates of $\vec{d }$ if abcd is a parallelogram and sketch the parallelogram

abcd in XY-plane. **/6marks**

1. Find the distance between two points nearest tenth**/3mks**
2. Given that A$ =\left(3, -2\right)$, B$ = \left(-5, 4\right)$ and C$\left(-1, -5\right)$
3. Find the equation of the straight line passing through points A and B.
4. Find the coordinates of point D if ABDC is a parallelogram.
5. Draw the parallelogram ABDC in Cartesian plane. **/6marks**

13. Show that the points A (0, –**2**), B(2, **4**) and C (–1, –5) are collinear**/3marks**

14. Calculate and simplify:$\left(1+\sqrt{2}\right)^{3}-\left(1-\sqrt{2}\right)^{3}$**/3marks**

15. Without using calculator,evaluate:$ 5.2×\left(3.75^{2}-1.25^{2}\right)$**/2marks**

**SECTION B (CHOOSE ONLY THREE QUESTIONS)**

1. The figure below shows the marks in percentage obtained by candidates in an English test

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 72 | 70 | 66 | 74 | 81 | 70 | 74 | 53 | 57 | 62 |
| 58 | 92 | 74 | 67 | 62 | 91 | 73 | 68 | 65 | 80 |
| 78 | 67 | 75 | 80 | 84 | 61 | 72 | 72 | 69 | 70 |
| 76 | 74 | 65 | 84 | 79 | 80 | 76 | 72 | 68 | 63 |
| 82 | 79 | 71 | 86 | 77 | 69 | 72 | 56 | 70 | 67 |
| 76 | 56 | 86 | 63 | 73 | 70 | 75 | 73 | 81 | 64 |

1. By arranging the data in classes of 50-54, 55-59, etc make frequency table.
2. Using your grouped data, calculate the mean and the median **/15marks**
3. Given that $ A\left(1,2\right)$ , $B\left(3,2\right)$and$C\left(2,1\right)$
4. Draw the triangle $ABC$on a Cartesian plane.
5. The image of $A$ by the translation is $A^{'}\left(-3,2\right)$. Find the coordinates of $B^{'}$ and $C^{'}$the images respectively of $B$ and $C$ by the same translation and draw triangle $A^{'}B^{'}C^{'}$ in the same Cartesian plane.
6. The image of $B$ by reflection is $B^{''}\left(-7,2\right)$

Determine the coordinates of $A^{''}$ and $C^{''}$images of $A$ and $C$ respectively by the same reflection.

1. Indicate the equation of the line of reflection and draw the triangle $A^{''}B^{''}C^{''}$ in the same Cartesian plane.

 Let the rotation of Centre$ O$, origin of axes, angle$ -90^{o}$. Find the coordinates of $A^{'''},B^{'''}$and $C^{'''}$image respectively of $A , B$ and $C$ by this rotation. Draw the triangle $A^{'''}B^{''}'C^{'''}$ in the same Cartesian plane**/15marks**

1. Given the polynomial function $P\left(x\right)=2x^{3}-5x^{2}-19x+42$
2. Find the quotient and the remainder of the division of $P(x)$ by $2x-7$
3. Factorise$P\left(x\right)$ completely.
4. solve the equation $P\left(x\right)=0$**/15marks**
5. a)In a school, students must take at least one of these subjects: Math, Physics and Chemistry. In a group of 50 students, 7 take all three subjects, 9 take Physics and Chemistry only, 8 take Math and Physics only and 5 take Math and Chemistry only. Of these 50 students $x $take Maths only, $x$ take Physics only and $x +3$ take Chemistry only. Draw the Venn diagram, find $x$ , and hence find the number taking Maths.

 b)A student of S3 deposited on his account an amount of 15000frw on a compound interest rate of 5% per annum (per year). Find his accumulated amount after 3 years.**/15marks**

1. a)Calculate the values of $m$ and $n$ if the polynomial $P\left(x\right)=2x^{3}+mx^{2}+nx+10$ has $-2$ as the root and also $2x-1$ is a factor of $P\left(x\right).$

b) Factorize completely $P(x)$

c)Deduce the roots of the equation $P\left(x\right)=0$**/15marks**

 **Good Luck !!!!!!!!!!!!!!!!!!!!**

 **MARKING GUIDE OF MATH DISTRICT EXAM S3 (TERM THREE, 2021)**

1. $\frac{3\frac{1}{2} – 1\frac{5}{6} × \frac{3}{11}}{1\frac{3}{4} +7\frac{ 2}{3} ÷3\frac{5}{6} } = \frac{\frac{7}{2} – \frac{11}{6} × \frac{3}{11}}{\frac{7}{4} + \frac{23}{3} ÷ \frac{23}{6}} =\frac{\frac{7}{2} – \frac{1}{2}}{\frac{7}{4} + \frac{23}{3} ×\frac{6}{23}} = $ **(2mark)**

 $\frac{3}{\frac{7}{4} +2} = \frac{3}{\frac{15}{4}} = \frac{3 ×4}{15} = \frac{4}{5}$ **(1mark)**

1. a)

 

 **(2mark)**

b) 

On the line number line



The integral values of x are: 2,3,4 and 5. **(2mark)**

 **(2mark)**

 **(1mark)**

1. For

|  |  |  |
| --- | --- | --- |
| x | 0 | 4 |
| y | 2 | 0 |

 **(1mark)**

|  |  |  |
| --- | --- | --- |
| x | 0 | -2 |
| y | 4 | 2 |

 **(1mark)**

 

 S= $\left\{(-1.33;2.67)\right\}$ **(2mark)**

1. $0 \leq 2x-3 \leq x+8$ $⟺2x-3 \geq 0$ and $2x-x \leq 8+3$ $⟺2x \geq 3$ and $x \leq 11$ $⟺x \geq \frac{3}{2}$ and $x \leq 11$ $⟺ \frac{3}{2}\leq x \leq 11$ **(2mark)** a) The prime numbers between $\frac{3}{2}$ and 11 are 3 , 5 , 7 , 11. **(1mark)** b) Even integers between $\frac{3}{2}$ and 11 are 4 , 6 , 8 , 10 **(1mark)**
2. $454\_{six}=4×6^{2}+5×6^{1}+4×6^{0}=144+30+4=178$ **(0.5mark)** $833\_{nine}=8×9^{2}+3×9^{1}+3×9^{0}=648+27+3=678$ **(0.5mark)** $221\_{three}=2×3^{2}+2×3^{1}+1×3^{0 }=18+6+1=25$  **(0.5mark)**

In base ten, $178+678-25=831$ **(0.5mark)**

 831 $÷$ 5 remainder

 166 1

 33 1

 6 3

 1 1

 0 1

$454\_{six}+ 833\_{nine}- 221\_{three}= 11311\_{five}$ **(1mark)**

1. $I=P×r×n$ **(1mark)** $r=\frac{I}{P×n}$$r=\frac{16000}{100000×2}$ **(1mark)**

 $r=0.08=8\%$

The annually rate is 8% **(1mark)**

1. $\frac{x}{6} = \frac{y}{4}$ and $2x +3y $ $=72$ **(1mark)**

$$4x =6y ⟹2x =3y ⟹3y+3y=72 ⟹6y=72$$

$ ⟹y=12 $ **(2mark)**

$2x+2x=72 ⟹4x=72 ⟹x=18 $ **(1mark)**

1. $P\left(x\right)=x^{4}-\left(1+\sqrt{2}\right)x^{3}+\left(1+\sqrt{2}\right)x^{2}-\left(1+\sqrt{2}\right)x+\sqrt{2}$
2. $P\left(\sqrt{2}\right)=\left(\sqrt{2}\right)^{4}-\left(1+\sqrt{2}\right)\left(\sqrt{2}\right)^{3}+\left(1+\sqrt{2}\right)\left(\sqrt{2}\right)^{2}-\left(1+\sqrt{2}\right)\sqrt{2}+\sqrt{2}$ **(1mark)**

$=4-2\sqrt{2}\left(1+\sqrt{2}\right)+2\left(1+\sqrt{2}\right)-\sqrt{2}-2+\sqrt{2} $ **(1mark)**

$=4-2\sqrt{2}-4+2+2\sqrt{2}-\sqrt{2}-2+\sqrt{2}$

$=0$ **(1mark)**

1. Use synthetic division:

 $\sqrt{2}\left|\begin{array}{c} \overline{\begin{array}{c}\begin{matrix}1&-\left(1+\sqrt{2}\right)&1+\sqrt{2} -\left(1+\sqrt{2}\right)\end{matrix}\\ \sqrt{2} -\sqrt{2} \sqrt{2} \\ \end{array}}\\ 1 -1 1 -1 \end{array}\right|\begin{array}{c}\sqrt{2}\\ \\-\sqrt{2}\\\\0\end{array}$

 $p\left(x\right)÷x-\sqrt{2}=x^{3}-x^{2}+x-1$ **(2mark)**

 **(1mark)**

 **(1mark)**

  **(1mark)**

b) abcd is a parallelogram then  where 



 **(1mark)**

 **(2mark)**

1.  **(3mark)**
2. $a) i) gradient m= \frac{4-(-2)}{-5-3}=\frac{6}{-8}=\frac{-3}{4} $ **(1mark)**







 **(1mark)**





 **(1mark)**



    **(1mark)**

**(2mark)**

 **(1mark)**

   **(2mark)**

 Therefore, the given points are collinear

$\left(1+\sqrt{2}\right)^{3}-\left(1-\sqrt{2}\right)^{3}=1+3\sqrt{2}+3\left(2\right)+2\sqrt{2}-\left(1-3\sqrt{2}+3\left(2\right)-2\sqrt{2}\right)$ $=7+5\sqrt{2}-\left(7-5\sqrt{2}\right)$ **(2mark)**

 $=10\sqrt{2}$ **(1mark)**

1. $5.2×\left(3.75^{2}-1.25^{2}\right)=5.2×\left(3.75+1.25\right)×\left(3.75-1.25\right)$ **(1mark)**

 $=5.2×5×2=65$. **(1mark)**

 **SECTION B**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Classes** | **f** | **x** | **Fx** | **CF** |
| 50-54 | 1  | 52 | 52  | 1 |
| 55-59 | 4  | 57 | 228  | 5 |
| 60-64  | 6  | 62 | 372  | 11 |
| 65-69  | 10  | 67 | 670  | 21 |
| 70-74  | 18  | 72 | 1296  | 39 |
| 75-79  | 9  | 77 | 693  | 48 |
| 80-84  | 7  | 82 | 574  | 55 |
| 90-94  | 3  | 87 | 261  | 58 |
| 95-99  | 2  | 92 | 184  | 60 |

 **(Two marks for each column i.e. 10mark for the table)**

$\overbar{X}=\frac{\sum\_{}^{}f×x}{n}=\frac{4330}{60}=72.17$ **(2mark)**

$Me=L+\frac{\frac{n}{2}-F}{f}×C$ **(1mark)**

$$L=69.5$$

$$F=21$$

$$f=18$$

$$C=74.5-69.5=5$$

$=69.5+\frac{30-21}{18}×5$

$=69.5+2.5$

$$=72$$

The median is 72**. (2mark)**



 **(1.5 marks for each triangle, 2marks a line d i.e. 8marks for whale diagram)** b) Vector of translation is $\left(\genfrac{}{}{0pt}{}{-3}{2}\right)-\left(\genfrac{}{}{0pt}{}{1}{2}\right)=\left(\genfrac{}{}{0pt}{}{-4}{0}\right)$ **(1mark)**

 Coordinates of B’: $\left(\genfrac{}{}{0pt}{}{3}{2}\right)+\left(\genfrac{}{}{0pt}{}{-4}{0}\right)=\left(\genfrac{}{}{0pt}{}{-1}{2}\right)$ **(1mark)**

 Coordinates of C’: $\left(\genfrac{}{}{0pt}{}{2}{1}\right)+\left(\genfrac{}{}{0pt}{}{-4}{0}\right)=\left(\genfrac{}{}{0pt}{}{-2}{1}\right)$ **(1mark)**

 c) i) Under reflection coordinates of A’’ are (-5,2) and coordinates of C’’ are (-6,1)

 ii) The line of reflection is given by $x=\frac{3+\left(-7\right)}{2}=\frac{-4}{2}=-2$

 The equation of the line is $x=-2$ **(1mark)**

d) Under a rotation of $-90°$ about the origin $\left(x,y\right)\rightarrow (y,-x)$

 $A^{'''}\left(2,-1\right) $ **(1mark)**

 $B^{'''}(2,-3)$ **(1mark)**

 $C^{'''}(1,-2)$ **(1mark)**

1. a) i) $2x^{3}-5x^{2}-19x+42$ divides by $2x-7$ using long division.

$2x^{3}-5x^{2}-19x+42$$2x-7$

 $-2x^{3}+7x^{2}$ $x^{2}+x-6$

 $2x^{2}-19x+42$

 $-2x^{2}+7x$

 $-12x+42$ **(5marks)**

 $12x-42$

 0

The quotient $q\left(x\right)=x^{2}+x-6$

The remainder R = 0

ii) $P\left(x\right)= \left(2x-7\right)\left(x^{2}+x-6\right)$ **(1mark)** $=\left(2x-7\right)\left(x^{2}+3x-2x-6\right)$ **(1mark)** $=\left(2x-7\right)\left[\left(x^{2}+3x\right)-\left(2x+6\right)\right]$ **(1mark)** $= \left(2x-7\right)\left(x+3\right)\left(x-2\right)$ **(1mark)**

iii) $P\left(x\right)=0$ $⟺\left(2x-7\right)\left(x+3\right)\left(x-2\right)=0$ **(1mark)**

 $\left\{\begin{array}{c}2x-7=0\\x+3=0\\x-2=0\end{array}\right. ⟺\left\{\begin{array}{c}x=\frac{7}{2}\\x=-3\\x=2\end{array}\right.$ **( 1mark for each value I.e. 3maks)**

1. a)

**(7mark)**

 **(2mark)**

The number of students taking Maths is  **(1mark)**

b) $A=P\left(1+\frac{r}{100}\right)^{n}$. **(2mark)**

P =15000F, R =5%, T =3 years **(1mark)**

A = 15000$ ×(1.05)^{3}frw$

A = 17,364,375 frw **(1mark)**

The accumulated amount of student after 3 years is 17,364,375 frws.

 **(1mark)**

1. $a) P\left(-2\right)=0 $

$⇒2\left(-2\right)^{3}+m\left(-2\right)^{2}+n\left(-2\right)+10=0⇒4m-2n=6 \left(1\right)$

$2x-1 $is a factor of $P\left(x\right)$ , **(3mark)**

then $P\left(\frac{1}{2}\right)=0 $

$⇒2\left(\frac{1}{2}\right)^{3}+m\left(\frac{1}{2}\right)^{2}+m\left(\frac{1}{2}\right)+10=0$

$⇒\frac{1}{4}+\frac{m}{4}+\frac{n}{2}+10=0$ **(3mark)**

$$⇒1+m+2n+40=0$$

$⇒m+2n=-41 \left(2\right) $

$\left(1\right)$ and $(2)$ gives :$\left\{\begin{array}{c}4m-2n=6\\m+2n=-41\end{array};\right.$ **(3mark)**

$5m=-35$, $m=-7$.

$m+2n=-41$ and $n=-17.$

 b)$P\left(x\right)=2x^{3}-7x^{2}-17x+10$

 $=\left(x+2\right)\left(2x^{2}-11x+5\right)$

 $=\left(x+2\right)\left[2x^{2}-10x-x+5\right]$

 $=\left(x+2\right)\left[2x\left(x-5\right)-\left(x-5\right)\right]$

 $=\left(x+2\right)\left(2x-1\right)(x-5)$ **(3mark)**

 C)$P\left(x\right)=0 ⇔\left(x+2\right)\left(2x-1\right)\left(x-5\right)=0$

 $⇔x+2=0 or$

 $ 2x-1=0or $

 $x-5=0 $

 $⇔x=-2 or x=\frac{1}{2} or x=5$ **(3mark)**